

Coulomb correlations and coherent charge tunneling in mesoscopic coupled rings

C. M. Canali¹, W. Stephan², L. Y. Gorelik¹, R. I. Shekhter¹ and M. Jonson¹

¹*Dept. of Applied Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden*

²*Max-Planck-Institut für Physik komplexer Systeme, Bayreuther Str. 40, D-01187 Dresden, Germany*

(1 October 1997)

We study the effect of a strong electron-electron (e-e) interaction in a system of two concentric one-dimensional rings with incommensurate areas A_1 and A_2 , coupled by a tunnel amplitude. For noninteracting particles the magnetic moment (persistent current) m of the many-body ground state and first excited states is an irregular function of the external magnetic field. In contrast, we show that when strong e-e interactions are present the magnetic field dependence of m becomes periodic. In such a strongly correlated system disorder can only be caused by inter-ring charge fluctuations, controllable by a gate voltage. The oscillation period of m is $\propto 1/(A_1 + A_2)$ if fluctuations are suppressed. Coherent inter-ring tunneling doubles the period when charge fluctuations are allowed.

PACS numbers: 71.27.+a, 73.20.Dx, 72.15.Nj

Persistent currents in quasi onedimensional mesoscopic rings [1] are a phase sensitive probe of the conductor equilibrium properties. The intriguing role played by e-e interactions in such a phase coherent phenomenon continues to be the source of intensive investigation. On one hand theoretical work [2] has shown that interaction does not influence the persistent currents in clean single-channel rings, in agreement with experiments on a single ballistic semiconductor ring with few channels [3]. On the other hand many other theoretical studies [4] have explored the possibility of linking the e-e interaction to the puzzlingly large current amplitudes found experimentally in disordered metallic (i.e. many-channel) rings [5]. Such amplitudes cannot be accounted for within a single-particle theory. No general consensus has been found yet on the effects of interaction in these systems.

As an example where e-e interaction is expected to play a distinctive role and cause interesting effects, we consider here a system of two concentric ballistic rings, coupled by a tunnel amplitude. The areas enclosed by the two rings are A_1 and A_2 respectively, and a perpendicular magnetic field B is present. Our aim is to investigate how strong interactions affect the many-body ground-state and the first excited states as a function of the applied magnetic field. We will show that the interaction creates a spatially ordered rigid structure, where the two rings are tied together and charge fluctuations tend to be suppressed. Consequently the flux dependence of the lowest energy levels becomes periodic with period $\propto 1/(A_1 + A_2)$, in contrast to the irregular aperiodic behavior of the free-particle case. By means of a gate voltage one can induce the coherent transfer of a single electronic charge from one ring to the other. As a result of the coherent charge fluctuations between the rings, the oscillation period of the lowest levels is doubled. Charge transfer induced effects in different mesoscopic samples have been recently studied by Büttiker and Stafford [6].

We choose a lattice model of two concentric one-dimensional rings with \mathcal{N} sites per ring (see Fig. 1), described by the following second quantized Hamiltonian

$$\begin{aligned} \mathcal{H} = & -t \sum_{i,p} e^{i2\pi\varphi_p/\mathcal{N}} c_{i,p}^\dagger c_{i+1,p} - \sum_i t_{12}(i) c_{i,1}^\dagger c_{i,2} + H.c. \\ & + \sum_{i,j,p,p'} V(|\mathbf{r}_{i,p} - \mathbf{r}_{j,p'}|) n_{i,p} n_{j,p'} \end{aligned} \quad (1)$$

The operators $c_{i,p}^\dagger$ and $c_{i,p}$ create and destroy a spinless electron at site i of ring p . ($i, j = 1, \dots, \mathcal{N}$ are site indexes and $p, p' = 1, 2$ are ring indexes). The term proportional to t describes the *intra-ring* hopping, affected by the magnetic field through the phases $\varphi_p = BA_p/\phi_0$, where $\phi_0 = ch/e$ is the flux quantum. The site-dependent amplitude $t_{12}(i)$ represents *inter-ring* tunneling. The last term is the translationally invariant e-e interaction. The number of electrons in each ring, N_p , is not a good quantum number if the rings are coupled. The expectation values $\langle N_p \rangle$ will however satisfy the constraint $\langle N_1 \rangle + \langle N_2 \rangle = N$; N is the total number of electrons in the system.

We first discuss the case of two decoupled [$t_{12}(i) = 0$] noninteracting rings. The single-particle spectrum of each ring is given by $\epsilon_p(k_i) = -2t \cos[(k_i + 2\pi\varphi_p/\mathcal{N})] = -2t \cos[(k_i + 2\pi H a_p/\mathcal{N})]$, with $k_i = 2\pi i/\mathcal{N}$, $i = 0, \pm 1, \dots, \pm \mathcal{N}/2$. Here we have introduced two independent, dimensionless parameters, H (magnetic field) and a_p (area of ring p). When $H = 1$ and $a_p = 1$ the flux enclosed by ring p is ϕ_0 . As H varies the levels move up and down; at some values of H they can cross. As a result, the particle at the Fermi energy can switch to an empty state of different “velocity”, causing a cusp in the magnetic field dependence of the ground state energy, $E_0(H)$. Cusps in $E_0(H)$ will cause jumps in the total magnetic moment of the ground state, $m(H) \equiv \partial E_0(H)/\partial H$. When N is even one can see that, in the ground-state, each ring of the system contains always $N/2$ particles. When flux-induced level crossing occurs, the particle at the Fermi energy will change place to an empty state of the *same* ring with opposite velocity. Therefore $m(H)$ will consist of the superposition of the contributions of two independent rings, each one oscillating with its own period $1/a_p$ as a function of H . If the areas of the two

rings are incommensurate, – throughout this work we have considered the values $a_1 = [\sqrt{5} - 1]/2 \approx 0.618$ and $a_2 = 1.5$ – the resulting total $m(H)$ is a bound but *aperiodic* function of H , with sharp jumps occurring at multiple of $1/a_p$, $p = 1, 2$. On the other hand when N is odd the relevant level crossings involve always two states belonging to *different* rings: an occupied state of the more populated ring is depleted in favor of an empty state of the other ring. In this case the aperiodic $m(H)$ contains jumps at values of H which are multiples of $1/(a_1 \pm a_2)$, corresponding to level crossings between states of different rings moving in opposite or equal direction respectively.

To first order in perturbation theory a small tunneling amplitude $|t_{12}(i)| \ll \Delta = 8t/\mathcal{N}$ (here Δ is the mean level spacing for each single ring) will couple states of different rings. If $t_{12}(i)$ is also site independent, the spectrum will hardly be modified and, in particular, the jumps in $m(H)$ will remain sharp, since they always involve crossing between levels of different momenta which cannot be coupled by a homogeneous hopping. However, if $t_{12}(i)$ is not homogeneous, states of different momenta can be coupled, causing a rounding of the sharp jumps in $m(H)$. If $|t_{12}(i)| \sim \Delta$ the energy levels of the two rings will be strongly hybridized and $m(H)$ will depend on the specific form of $t_{12}(i)$.

In order to see how strong e-e interactions modify this scenario we have resorted to exact numerical calculations on small systems, using the Lanczos algorithm to compute the many-body ground state and the first low-lying excitations. We have considered systems with $\mathcal{N} = 11$ sites per ring and $N = 4, 5, 6, 7, 8$ particles. We will first discuss the case of a short range interaction, coupling two nearest-neighbor sites (i, p) and $(i + 1, p)$ in the same ring with matrix element V_{11} , and opposite sites $(i, 1)$ and $(i, 2)$ in different rings with matrix element V_{12} . The relevant energy parameter, indicating the relative strength of the interaction, is $V_{pp'}/t$. Plots of the magnetic moment for systems of $N = 6, 7, 8$ particles interacting with $V_{11} = 6.5t$, $V_{12} = 6.3t$ are shown in Fig. 2. The effect of the interaction is visibly spectacular, particularly when N is even: the aperiodic H -dependence of $m(H)$ of the noninteracting case is replaced by a perfectly *periodic* function, oscillating with period $H_0 = 1/(a_1 + a_2)$, exactly as we would have for a single ring with area $a_{\text{tot}} = (a_1 + a_2)$. The periodic $m(H)$ develops smoothly upon increasing the interaction and once it is reached it becomes independent of the interaction strength. The amplitude of $m(H)$ in the strongly interacting regime is approximately one half of the amplitude of the noninteracting case. When N is even, the persistent current $I(H) \equiv -cm(H)$ is diamagnetic, opposite to the single-ring case. As shown in Fig. 2(c), an inter-ring hopping $t_{12}(i)$ of any kind does not spoil this result, even when its magnitude is comparable to the intra-ring hopping t . Moreover, not only is the ground-state energy a periodic function of H , but so are the first low-lying excited states. In particular, the low-energy sector of the many-

body spectrum is qualitatively identical to the single-particle spectrum in one ring. We can also evaluate the particle number in each ring for each many-body state by computing the corresponding wavefunction. The calculation shows that in the strongly interacting regime all periodic eigenstates have $N_1 = N_2 = N/2$ for any value of H when the rings are decoupled. Inter-ring charge redistributions, causing H -dependent deviations from the $(N_1 = N/2, N_2 = N/2)$ configuration, are suppressed here. Charge fluctuations occur however in the higher aperiodic states; these are separated from the ordered low-energy sector by an energy gap that scales with $V_{pp'}/t$. A non-zero $t_{12}(i) \sim \Delta \ll V_{pp'}$ cannot connect these two sectors of the spectrum. Therefore the low-lying states remain periodic with $\langle N_1 \rangle \approx \langle N_2 \rangle \approx N/2$, even when the rings are coupled.

Strong intra-ring interaction is known to generate Wigner-crystal-like ground states in a single ring [7]. It has also been shown that inter-ring interaction causes a nondissipative Coulomb drag [8] that affects qualitatively the persistent currents in each ring [9]. Our exact results suggest that strong inter-ring interaction glues together the two stiff Wigner crystals created by the intra-ring interaction, resulting in a rigid structure that rotates as a single solid body under the effect of the magnetic field. Based on this picture, we can easily see why the oscillation period of the total magnetic moment of such a structure is $1/(a_1 + a_2)$. Indeed the Hamiltonian of the two frozen crystals glued together, each containing $N/2$ particles, is found to be

$$\mathcal{H} = \frac{\hbar^2}{2\mathcal{I}} \left[-i \frac{\partial}{\partial \theta} - \frac{N}{2} H(a_1 + a_2) \right]^2 \quad (2)$$

where $-i \hbar \frac{\partial}{\partial \theta}$ is the canonical momentum conjugated to θ , the rotation angle of the solid around an axis passing through the center of the rings; $\mathcal{I} = m_e \frac{N}{2\pi} (A_1 + A_2)$ is the moment of inertia. In zero flux, the eigenstates $\Psi_n(\theta) = \exp(in\theta)$ must satisfy the boundary condition $\Psi_n(\theta) = \Psi_n(\theta + 2\pi/(N/2))$ due to the ordered crystal structure. This immediately implies the announced $1/(a_1 + a_2)$ periodicity as a function of H .

The $m(H)$ for odd N [see Fig 2(b)] becomes also periodic in presence of a strong interaction, with the same period H_0 . One significant difference with respect to even N is the shift of $m(H)$ by $H_0/2$: the system is now paramagnetic.

We have also studied a more realistic unscreened long range interaction coupling all the sites with matrix element $V(|\mathbf{r}_{i,p} - \mathbf{r}_{j,p'}|) = V_c R_1/|\mathbf{r}_{i,p} - \mathbf{r}_{j,p'}| = V_c \sqrt{a_1}/[a_p + a_{p'} - 2\sqrt{a_p a_{p'}} \cos(2\pi(i-j)/\mathcal{N})]^{1/2}$. The dimensionless interaction constant is $\alpha = V_c R_1/\hbar v_F = (V_c/t)[\mathcal{N}/4\pi \sin(k_F)]$. In the strong interaction limit ($\alpha > 5$) the Hamiltonian Eq. [1] has an unphysical ground state where all the particles are located in the outer ring. Obviously a realistic Hamiltonian should contain the contribution of a positive background preventing the deple-

tion of the inner ring. This is usually done by replacing $n_{i,p}$ by $n_{i,p} - K$ in Eq. [1], where K represents the average positive jellium which equals the average electron density. Such a replacement is formally equivalent to add in the Hamiltonian Eq. [1] a potential difference between the rings, $H_g = \sum_{i,p} \epsilon_p n_{i,p}$. Since we also want to consider the effect of a real external gate voltage $V_g = \epsilon_1 - \epsilon_2$ we will assume that the background is incorporated in V_g , which we will take as a parameter. Typically a voltage difference $V_g \approx -V_c/2$ ensures an almost equal population in the two rings. In Fig. 3 we plot $m(H)$ for $N = 5, 6$ particles interacting with a Coulomb force of intensity $V_c = 10t$, for two values of V_g . Let us consider first $N = 6$ and $t_{12} = 0$ (black line in Fig. 3(a)(b)). For $V_g = -0.75 V_c$, $m(H)$ is perfectly periodic with the expected period $1/(a_1 + a_2)$. On the other hand imperfections in the periodic pattern appear for $V_g = -0.5 V_c$. Note in particular the small glitches at $H = 0.3, 1$ and the splitting of the peak at $H = 3$ in Fig. 3(a). In fact the calculation of the ring occupancy shows that while for $V_g = -0.75 V_c$ the electronic configuration $(N_1 = 3, N_2 = 3)$ is stable for all values of H , when $V_g = 0.5 V_c$ the two configurations $(N_1 = 3, N_2 = 3)$ and $(N_1 = 2, N_2 = 4)$ alternate in the ground state. The aperiodic jumps in Fig. 3(a) correspond to a switch from one configuration to another as H varies. It is instructive to look at the H dependence of the lowest four energy levels, plotted in Fig. 4, and their corresponding ring occupancy. Fig. 4(a) shows that when $V_g = -0.5 V_c$ these four levels are often two by two degenerate, the degeneracy being caused by the presence of the two electronic configurations $(N_1 = 3, N_2 = 3)$ and $(N_1 = 2, N_2 = 4)$ mentioned above. V_g is not yet strong enough to enforce a fixed electronic configuration in the two rings. As a result the levels are not completely periodic and they are sensitive to inter-ring tunneling— see grey line of Fig 3(a). However if we increase V_g up to $0.75 V_c$, the configuration $(N_1 = 3, N_2 = 3)$ is locked in the first three states. Charge redistribution becomes possible only starting from the fourth level, with the appearance of the $(N_1 = 2, N_2 = 4)$ configuration at some values of H . The important consequence is the ordering of the first three levels, which is robust against tunneling between the rings, as shown in Fig. 3(b) and Fig. 4(c).

The example of 6 particles is representative of what should happen in a real mesoscopic system with $N_1 \sim N_2 \sim 50$, where gate- and flux-induced charge redistribution between the rings amounts to small changes in their electron concentration. If the gate voltage enforces an almost equal population and prevents charge fluctuations between the rings as H varies, then the e-e Coulomb interaction can fully develop the ordering effect.

The presence, at some values of V_g , of two quasi-degenerate states that differ by one single electron being moved from one ring to the other gives rise to the possibility of coherent tunneling [6] in presence of strong correlation. This effect is best illustrated for odd N when the swapping of one electron between the rings gener-

ates the two configurations $(N_1 = \frac{N+1}{2}, N_2 = \frac{N-1}{2})$. In this case a homogeneous t_{12} coupling the two rings allows the swapping electron to occupy an hybridized orbital involving two opposite sites of the two rings with equal probability. Since the particles are indistinguishable all of them can be coherently involved in this process. This means that all the N particles will spend half of their time on either ring, that is $\langle N_1 \rangle \approx \langle N_2 \rangle \approx N/2$, which is equivalent to having N particles on a single ring of area $(a_1 + a_2)/2$. The period of the persistent current of such “dynamically-glued” Wigner crystals will be $2H_0 = 2/(a_1 + a_2)$, twice as large as the previous case, where charge fluctuations were totally suppressed. The numerical results for $N = 5$ shown in Fig. 3(c) confirms fully this prediction: the presence of a homogeneous t_{12} coupling coherently the quasi-degenerate configurations $(N_1 = 2, N_2 = 3)$ and $(N_1 = 3, N_2 = 2)$ doubles the oscillation period of $m(H)$. Furthermore also the amplitude of $m(H)$ is factor of 2 larger than when $t_{12} = 0$. A similar result holds approximately for $N = 6$, see Fig. 3(b), but now the oscillating period must be generalized to $N/(\langle N_1 \rangle a_1 + \langle N_2 \rangle a_2)$. For $N = 6$ the computed averaged occupations are $\langle N_1 \rangle \approx 2.5$, $\langle N_2 \rangle \approx 3.5$ and the resulting period is a little smaller than $2/(a_1 + a_2)$. When $N \gg 1$ we have $\langle N_1 \rangle/N \approx \langle N_2 \rangle/N \approx 1/2$ and we obtain the result that holds exactly for $N = 5$.

Our calculations show that spectrum ordering is already well developed for $\alpha \approx 5$. Experimentally, GaAs-Al/GaAs heterostructures with $\alpha > 1$ are available. Thus the effects that we have found might be seen in a system of two semiconductor rings. Our results might also be connected to the halving of the oscillation period of the Aharonov-Bohm effect in Quantum Hall edge states around a semiconductor antidot [10]. The experiment in Ref. [10] showed that when just two edge states encircled the antidot at approximately the same radius, two sets of AB oscillations seemed to lock together, but out of phase, giving an apparent $(h/e)/2$ oscillation. This seems to correspond to a double-ring system with $A_1 = A_2$, with periodicity proportional to $1/2A_1$.

In conclusion, we have shown that e-e interaction orders the flux dependence of the low-energy spectrum of a two-ring system. When strong e-e correlations are present, disorder can only be induced by inter-ring charge redistribution controllable through an external gate.

We thank I. V. Krive for useful discussions. Financial support from NFR and TFR is gratefully acknowledged.

-
- [1] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983); M. Büttiker, Phys. Rev. B **32**, 1846 (1985).
 - [2] A. J. Legget, in *Granular Nanoelectronics*, edited by D. K. Ferry (Plenum Press, New York, 1991), p. 297. D. Loss, Phys. Rev. Lett. **69**, 343 (1992).

- [3] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2024 (1993).
- [4] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990); A. Schmid, *ibid.* **66**, 80 (1991); B. L. Altshuler, Y. Gefen, and Y. Imry, *ibid.* **66**, 88 (1991); M. Abraham and R. Berkovits, *ibid.* **70**, 1509 (1993).
- [5] L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990); V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Galager and A. Kleinsasser, *ibid.* **67**, 3578 (1991).
- [6] M. Büttiker and C. A. Stafford, Phys. Rev. Lett. **76**, 495 (1996).
- [7] H. J. Schultz, Phys. Rev. Lett. **71**, 1864 (1993); I. V. Krive *et al.*, Phys. Rev. B **52** 16451 (1995).
- [8] A. G. Rojo and G. D. Mahan, Phys. Rev. Lett. **68**, 2074 (1992).
- [9] T. V. Shahbazyan and S. E. Ulloa, unpublished.
- [10] C. J. B. Ford *et al.* Phys. Rev. B **49**, 17456 (1994).

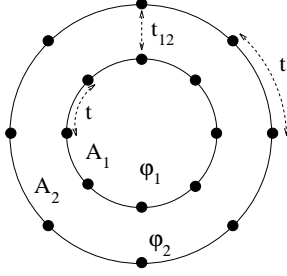


FIG. 1. Schematic picture of two concentric rings with enclosed areas A_1 and A_2 . The black dots are the lattice sites.

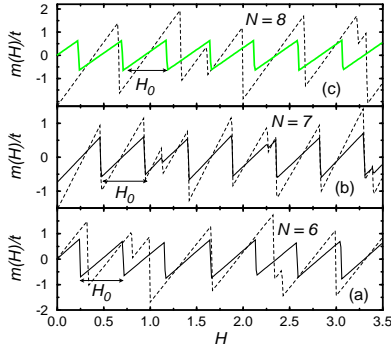


FIG. 2. Magnetic moment vs H for the sample of Fig. 1 with $\mathcal{N} = 11$ sites per ring. The normalized areas are $a_1 = [\sqrt{5} - 1]/2 \approx 0.618$ and $a_2 = 1.5$ respectively. The total number of particles is: (a) $N = 6$, (b) $N = 7$ and (c) $N = 8$. *Aperiodic* dashed curves: decoupled rings [$t_{12}(i) = 0$], noninteracting particles. *Periodic* black lines: $t_{12}(i) = 0$, particles interacting via a short range force; the oscillation period, is $H_0 = 1/(a_1 + a_2) \approx 0.47$. *Periodic* gray line in (c): $t_{12}(i) = -t/2 \cos^2(3\pi i/2\mathcal{N})$, interacting particles.

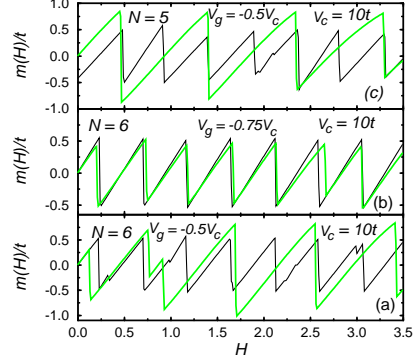


FIG. 3. Magnetic moment vs H for particles interacting with a Coulomb potential of intensity $V_c = 10t$. V_g is the voltage between the rings. Black lines: $t_{12}(i) = 0$; Gray lines: $t_{12}(i) = -t/2$. (a) $N = 6$ particles, $V_g = -0.5 V_c$. (b) $N = 6$, $V_g = -0.75 V_c$. (c) $N = 5$, $V_g = -0.5 V_c$.

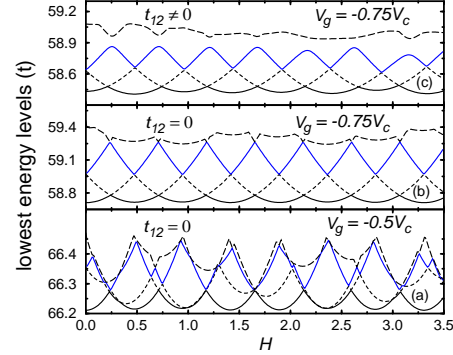


FIG. 4. Energy vs H of the ground state and the first three excited states for a system of $N = 6$ particles, interacting with a Coulomb potential of intensity $V_c = 10t$. V_g is the voltage between the rings, (a) $t_{12} = 0$, $V_g = 0.5 V_c$. (b) $t_{12} = 0$, $V_g = -0.75 V_c$. (c) $t_{12}(i) = -t/2$, $V_g = -0.75 V_c$.